Task: Jenna's Homework Problem — 5th Grade

Jenna completed 2/3 of the math problems on her homework during her stay at Fun Company after school. Afterward, she finished 1/8 of the total problems at home before she went to bed. A.) If 16 problems represent 2/3 of her math homework, how many problems were on her homework assignment in all? B.) What combined fraction of her homework, counting the fraction she did at Fun Company and the fraction she did at home, did Jenna finish before she went to bed? C.) What fraction of the total assignment does she still need to do before class tomorrow and how many problems is this?

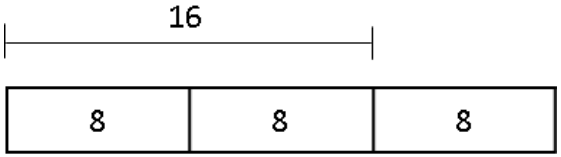
Teacher Notes:

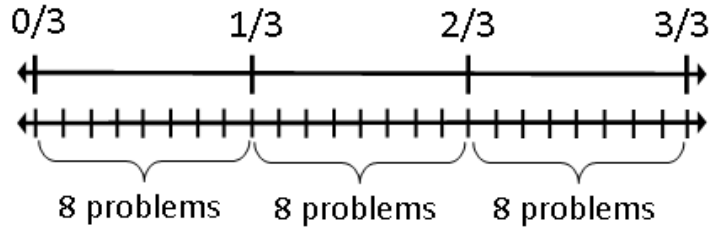
Teachers may want to use this task as an introduction to adding fractions with unlike denominators. Using manipulatives to model this situation will help students understand the connection between the number of problems on the assignment and the fractions of the whole assignment. Using manipulatives will also highlight the need for finding a common denominator.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

The concept of *unit* is fundamental to the interpretation of rational numbers.
One interpretation of a rational number is as a part-whole relationship.
One interpretation of a rational number is as a ratio.
Whole number conceptions of *unit* become more complex when extended to rational numbers.
Any rational number can be expressed as a fraction in an infinite number of ways.
The interpretations of the operations on rational numbers are essentially the same as those on whole numbers, but some interpretations require adaptation, and the algorithms are different.

Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
A. The whole assignment can be broken into three equal parts. Since 2 of the three equal parts represents 16 problems, then we divide 16 by 2 to see how many problems are in 1 part. (16 ÷ 2 = 8) This means there are 8 problems in each of the three sections.  3 x 8 = 24 problems Now multiply 8 x 3 to see the total number of problems on the assignment. (8 x 3 = 24) There are 24 problems on the whole assignment. OR We can use a number line to represent the whole assignment and break it into 3 equal parts. We can add another number line of the same length below the previous one and evenly divide 16 within the space taken up by 2 of 3 sections from the top number line.	**Assessing** A. • How did you know to divide 16 by 2? • Why did you multiply 8 by 3? • If we want 3 parts, why not divide 16 by 3? • How did you use the denominator to help you solve this part of the problem? • How did you use the numerator to help you solve this part of the problem? • Should there be the same number of problems in each third? Why, why not? • Why did you break the number line into 3 equal parts? • Would it matter if your number lines were not the same length? Why, why not? **Advancing** A. • How would you answer this if the number of problems given as 2/3 of the assignment was not divisible by 2? • How could you change the problem to account for Jenna working part of a problem, but not the whole problem? • Can you represent this problem in a different way?

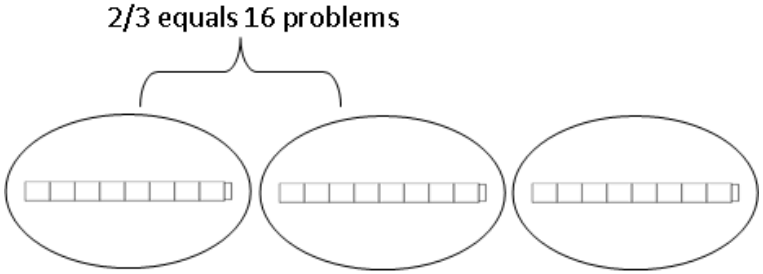


3 x 8 = 24 problems

This will show that there are 8 tick marks in each third so there will be 8 more tick marks in the last third for a total of 24 tick marks. Each tick mark on the lower number line represents 1 problem so there are 24 problems on the whole assignment.

OR

We could use 16 unifix cubes to represent 2/3 of the whole assignment. We can break the 16 cubes into 2 equal parts to represent 2 of the three equal groups we should have.



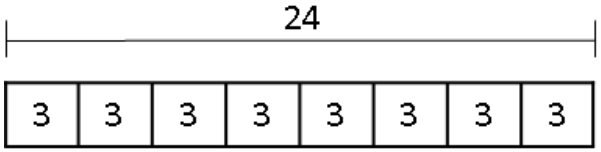
3 x 8 = 24 total problems

We see that there are 8 cubes in each group so we must add 8 more to have 3 equal groups as necessitated by the denominator. This gives a total of 24 cubes, or 24 problems on the assignment.

OR

16 ÷ 2 = 8 then 8 X 3 = 24. There are 24 problems on the whole assignment.

B. Since we know that 2/3 of the assignment is represented by 16 problems and there are 24 problems in all, we can divide 24 by 8 to get the number of problems in 1/8 of the assignment. (24 ÷ 8 = 3)



1/8 of 24 = 3 problems

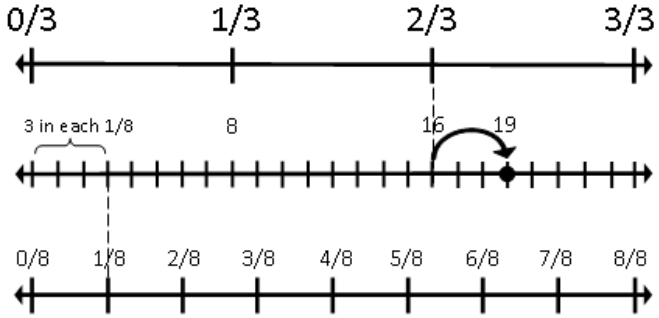
16 + 3 = 19 problems

19/24 problems have been completed

Now we can add the number of problems that represents each fraction then use that to write the ratio of problems completed to the total number of problems on the assignment. (16 + 3 = 19) Jenna has completed 19/24 of the assignment before bed.

OR

We can now add a third number line below the first two. This number line should be the same length as the other two, but should be broken into 8 equal pieces.



16 + 3 = 19 problems completed

2/3 + 1/8 = 19/24 problems completed

We can now see that there are 3 tick marks from the middle number line in the space for each of the 8 sections of the

Assessing

B.

- Why did you divide 24 by 8?
- How did you know to add 3 to 16?
- Which number line represents the total number of problems in the assignment?
- What do the tick marks stand for on the middle number line?
- How are the top and middle number lines related and what do they tell you about the problem?
- How are the middle and bottom number lines related and what do they tell you about the problem?
- Why did you multiply 2/3 by 8/8?
- Why did you multiply 1/8 by 3/3?
- How is the common denominator related to the total number of problems in the assignment?

Advancing

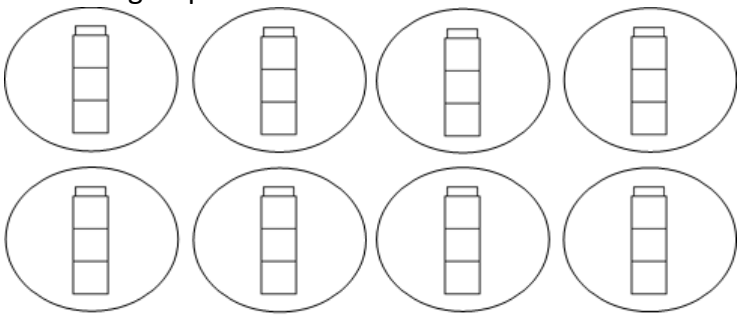
B.

- Could you find the fraction of the total assignment Jenna had done before bed if you were not given the number of problems that represented 2/3 of the whole? If so, how? If not, why not?
- Can you represent this problem in a different way?

bottom number line. This means there are 3 problems in 1/8 of the assignment. In order to get the combined number of problems, we can start at the 16 on the middle number line and jump 3 tick marks to get a total of 19. Since there were 24 total tick marks on this number line, we can write the ratio of completed problems to the total problems as 19/24. Jenna has completed 19/24 of the assignment before bed.

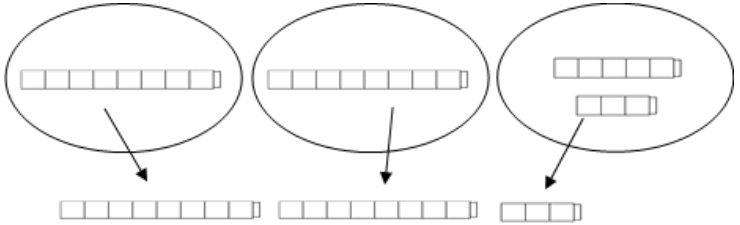
OR

We can now separate the 24 cubes into 8 equal groups with 3 in each group.



1/8 equals 3 problems

So we know that 3 problems represent 1/8 of the whole assignment. Now we can regroup the 24 cubes into three even groups. We can put two of those groups together to get the 16 we started with and take 3 from the third group and add it to the 16 to represent adding 2/3 of the whole to 1/8 of the whole. This will give us 19 cubes out of the 24 total.



2/3 + 1/8 = 19/24

Thus, 2/3 + 1/8 = 19/24. Jenna has done 19 problems and

this is 19/24 of the assignment.

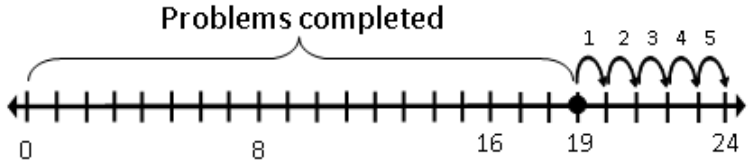
OR

2/3 + 1/8 = (2/3 x 8/8) + (1/8 x 3/3) = 16/24 + 3/24 = 19/24
Jenna has done 19/24 of the whole assignment, which is 19 problems out of the 24.

C. Again we can use the total number of problems on the assignment to help us get the ratio of unfinished problems to the total number of problems. Since there are 24 problems in all and Jenna has completed 19, we can subtract to find how many problems she still needs to do. (24 – 19 = 5). Jenna still needs to do 5 problems. Five problems represent 5 out of 24 so the fraction of the assignment Jenna still needs to complete is 5/24.

OR

We can use the middle number line and start at 19 to represent the number of problems Jenna has completed. We can then count the tick marks from there to the end of the number line to determine how many problems she has left to do.



5 problems are not completed

5/24 problems still need to be

We can then write that as a ratio of problems left to do to total problems. Jenna still needs to do 5 more problems or 5/24 of the whole assignment.

OR

We can use the 24 cubes that have been broken into one set of 19 and the remaining set of 5 cubes to show that there are 5 problems left to do and that represents 5/24 of the whole assignment.

Assessing

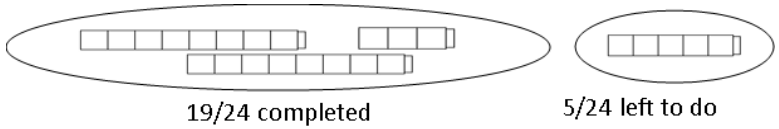
C.

- Why did you subtract 19 from 24?
- What does the 5 represent?
- Why did you subtract 19/24 from 1?
- What does the 1 represent?
- Why/how did you change 1 to 24/24?

Advancing

C.

- Can you represent this in a different way?
- If Jenna worked 1/8 of the problems at Fun Company and 2/3 of them at home, would the answer be the same? Why/why not?

 OR 1 – 19/24 = 24/24 – 19/24 = 5/24 Thus, Jenna still needs to do 5 more problems and this is 5/24 of the whole assignment.	
Possible Student Misconceptions	
Students may think that 16 problems represent the whole.	Are there 16 problems in the whole assignment? Do the 16 problems represent part of the assignment or the whole assignment?
Students may think the task is referencing 1/8 of the remaining problems rather than 1/8 of the whole.	Is there a word in the sentence that will help you decide if Jenna did 1/8 of the whole assignment at home or 1/8 of what was left to do after she had already done the part at Fun Company?
Students may use the rectangle model to represent this by sectioning the rectangle into 3 columns and 8 rows then shading in 2 of the columns and 1 of the rows and counting the shaded blocks and the total blocks. In doing this, they may fail to realize that two of the blocks are shaded each time, thus requiring that they be counted twice. (Students may confuse the area model for multiplication with a model needed for addition.)	How many blocks are shaded in all? Were there any blocks shaded twice? If so, how should we deal with those? (What operation do we usually use a rectangular area to model? Does this task require that operation?)
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	If 16 problems represent 2/3, how many problems are in 1/3? How many problems are in 3/3? How many problems are in 1/8? What operation can you use to combine the fractions?
If students finish early….	Have them choose their favorite solution path and write it on chart paper to present to the class. With a partner, each person tries to explain the other person's solution. Have them write another problem of this type to give to the class. They must also provide you with a solution so the class may check their work. Have them work a problem like this, but don't give any information about the number of problems on the assignment.

Discuss/Analyze

Whole Group Questions

Key Understandings:

- 2/3 of the assignment is represented by 16 problems; therefore there are 24 problems in the whole assignment.
- Once you know the number of problems in 1/3, 2/3 and 1/8 of the assignment, you can find the total number of problems Jenna did.
- You can also find the fraction of the problems Jenna completed and still needs to complete if you understand the concept of ratio.
- You can't add these fractions without a common denominator.
- The total number of problems on the assignment represents the common denominator.
- The whole assignment represents 1 unit.
- The 24 problems represent 1 unit.
- The denominator tells how many equal groups are in the whole.
- The same whole can be split into different numbers of equal groups. (i.e. 24 can be split into 8 groups of 3; 4 groups of 6; 2 groups of 12)
- In order to combine parts, they must be the same kind of part. (i.e. 8ths with 8ths; 24ths with 24ths)
- To find the remainder, you must subtract the part from the whole.
- A whole can be represented in multiple ways. (i.e. 1; 3/3; 8/8; 24/24)

Questions:

- How many different ways is the whole assignment represented in this task?
- How can you use what you know about part of the assignment to get the whole?
- 19/24 represents the ratio of what to what?
- 5/24 represents the ratio of what to what?
- How many ways can you represent 2/3 or 1/8 of the assignment?
- Can you add 2/3 with 1/8 without changing them? Why or why not?
- What must be true in order for you to be able to add two fractions? Why?
- If I know that Jenna did 19 problems out of 24, what should I subtract from what to find out the fraction of problems she still needs to do?
- What does 24-19 tell me about Jenna's homework? How is that different from what 1-19/24 tells me?