

<b>Task: Car Lot</b> <span style="float: right;">6<sup>th</sup> Grade</span>	
<p>The local car lot has 8 cars and 5 trucks. The owners are going to buy enough cars and enough trucks to make the ratio of cars to trucks 3:2.</p> <p>A) How many cars and how many trucks should the owners buy?</p> <p>B) Is there more than one way to buy enough cars and trucks to make the ratio of cars to trucks 3:2?</p> <p>C) Describe any patterns you see in solving this problem.</p>	
<b>Common Core State Standards for Mathematical Content</b>	<b>Common Core State Standards for Mathematical Practice</b>
<p>6.RP.A.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</i></p> <p>6.RP.A.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p style="padding-left: 20px;">a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>6.EE.A.2. Write, read, and evaluate expressions in which letters stand for numbers.</p> <p style="padding-left: 20px;">a. Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as <math>5 - y</math>.</i></p> <p>6.EE.C.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</i></p>	<p>Mathematical Practices</p> <ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>
<b>Essential Understandings</b>	
<ul style="list-style-type: none"> <li>• Reasoning with ratios involves attending to and coordinating two quantities.</li> <li>• Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.</li> <li>• Ratios are often used to make “part-part” comparisons, but fractions are not.</li> <li>• Using variables permits representing varying quantities. This use of variables is particularly important in studying relationships between</li> </ul>	

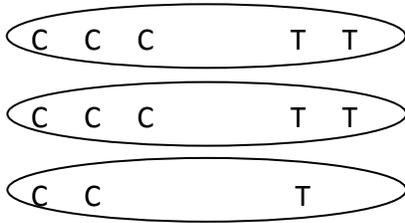
varying quantities.

**Explore Phase**

**Possible Solution Paths**

Drawing a Picture

A) Students can represent 8 cars and 5 trucks. Since the ratio should be 3:2, they can circle groups of 3 cars and 2 trucks.



The third group is missing 1 car and 1 truck, so the owners need to purchase 1 additional car and 1 additional truck.

B) Students can extend the reasoning from part A by including additional groups of 3 cars and 2 trucks. Additional answers include: 4 cars and 3 trucks; 7 cars and 5 trucks; 10 cars and 7 trucks; etc.

C) Patterns: The number of cars purchased should be 1 more than a multiple of 3 ( $3x + 1$ ); the corresponding number of trucks should be 1 more than the same multiple of 2 ( $2x + 1$ ).

Making a Table

A) Students can represent a table showing ratios equivalent to 3:2

**Assessing and Advancing Questions**

Part A)

Assessing Questions

Why did you choose to group the cars and trucks the way you did?

Advancing Questions

Are there other ways to represent the ratio 3:2? How would those other ways be helpful?

Part B)

Assessing Questions

How can you show that there is more than one way to make the ratio of cars to trucks 3:2?

Advancing Questions

Are there other ways to represent the ratio 3:2? How would those other ways be helpful?

Part C)

Assessing Questions

How do you know your pattern is correct?

Is there a relationship between the pattern for the number of cars the owners should buy and the pattern for the number of trucks the owners should buy?

Advancing Questions

Is there a pattern to the number of cars the owners should buy? The number of trucks?

How can you describe those patterns using a variable?

Part A)

Assessing Questions

How did you decide how many cars and how many trucks the owners

to determine the number of cars and trucks the owners need to purchase.

Cars:Trucks = 3:2		
Number of Cars Needed	Number of Trucks Needed	Notes
3	2	The lot already has 8 cars and 5 trucks—more than these numbers.
6	4	The lot already has 8 cars and 5 trucks—more than these numbers.
9	6	The lot already has 8 cars and 5 trucks—the owners can purchase 1 car and 1 truck.

B) Students can extend the table begun in part A.

Cars:Trucks = 3:2		
Number of Cars Needed	Number of Trucks Needed	Notes
3	2	The lot already has 8 cars and 5 trucks—more than these numbers.
6	4	The lot already has 8 cars and 5 trucks—more than these numbers.
9	6	The lot already has 8 cars and 5 trucks—the owners can purchase 1 car and 1 truck.
12	8	The lot already has 8 cars and 5 trucks—the owners can purchase 4 cars and 3 trucks.
15	10	The lot already has 8 cars and 5 trucks—the owners can purchase 7 cars and 2 trucks.
etc.	etc.	

should buy?

Advancing Questions

Are there other ways to represent the ratio 3:2? How would those other ways be helpful?

Part B)

Assessing Questions

How can you show that there is more than one way to make the ratio of cars to trucks 3:2?

Advancing Questions

Are there other ways to represent the ratio 3:2? How would those other ways be helpful?

Part C)

Assessing Questions

How do you know your pattern is correct?

What does the variable in your pattern represent?

Is there a relationship between the pattern for the number of cars the owners should buy and the pattern for the number of trucks the owners should buy?

Advancing Questions

Is there a pattern to the number of cars the owners should buy? The number of trucks?

How can you describe those patterns using a variable?

C) Since the number of cars bought increases by 3 at each step, we know that the pattern should have 3 (a variable) in it. The sequence describing the number of cars bought is an arithmetic sequence that begins with 1, so the pattern is  $3x + 1$ , where  $x$  is a whole number. Similarly, the pattern for the number of trucks is  $2x + 1$ . It is important to note that to find an answer to problem A, the same value for  $x$  should be used in both patterns.

Number of Cars Bought	Pattern for the number of cars	Number of Trucks Bought	Pattern for the number of trucks
1	$3(0) + 1$	1	$2(0) + 1$
4	$3(1) + 1$	3	$2(1) + 1$
7	$3(2) + 1$	5	$2(2) + 1$
10	$3(3) + 1$	7	$2(3) + 1$

### Possible Student Misconceptions

Students may not think a solution is possible since 3:2 implies 3 cars and 2 trucks, and the car lot already has 8 cars and 5 trucks.

#### Assessing Questions

What does the ratio 3:2 mean in the problem? What is the ratio of cars to trucks now?

#### Advancing Questions

Are there other ways to write the ratio 3:2? How could these other ways help you?

Students may confuse 3:2 as the ratio of trucks to cars.

#### Assessing Questions

What does the ratio 3:2 mean in the problem? What does the 3 represent and what does the 2 represent in the ratio?

#### Advancing Questions

Are there other ways to write the ratio 3:2? How do you know which number in your ratios represents the number of cars and which represents the number of trucks? How would this help?

Patterns: Students may describe the pattern for the number of cars bought as  $3x$  or as  $x + 3$  rather than  $3x + 1$ . Similar issues arise for the pattern for the number of trucks bought.

#### Assessing Questions

Tell me what pattern you see in your answers. How can you describe this pattern?

What does the variable represent in your pattern?

	<p><u>Advancing Questions</u>          What kinds of numbers can you use in place of the variable in your expression? Do your answers from your expression match the answers you found in part B?</p> <p>How would you decide how many cars the owners should buy if they buy 5 trucks? How do you know? What number do you use for x in your expression to determine the number of cars?</p>
<b>Entry/Extensions</b>	<b>Assessing and Advancing Questions</b>
If students can't get started....	<p><u>Assessing Questions</u>          Let's look at what is already on the car lot. How many cars and how many trucks do you have? What is the ratio of cars to trucks now? Why?</p> <p><u>Advancing Questions</u>          How can you change that ratio? What would you need to do to change that ratio?</p>
If students finish early....	<p><u>Assessing Questions</u>          Show me how you determined the expressions for the patterns you found.</p> <p>What kinds of numbers can you use in your expressions for the variable?</p> <p>Is the variable in your expression for the number of cars related to the variable in your expression for the number of trucks?</p> <p><u>Advancing Questions</u>          Suppose the ratio of trucks to cars was 3:2. How would this change your answers? Would you have different patterns? Why or why not?</p>
<b>Discuss/Analyze</b>	
<b>Whole Group Questions</b>	
What strategies did you use to solve this problem? How did you know your strategy would work?	
How are ratios in part B similar to what you know about fractions? How are the ratios different than what you know about fractions?	
What kinds of numbers can you use in your expressions for the variable? Is the variable in your expression for the number of cars related to the variable in your expression for the number of trucks?	

